

## **SUPPORTING SPECIAL NEEDS STUDENTS IN DRAWING MATHEMATICAL REPRESENTATIONS**

### **Technology in Practice Strand**

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### **1. ABSTRACT**

The *INK-12: Interactive Ink Inscriptions in K-12* project, a collaboration between MIT and TERC, has been investigating the use of a pen-based wireless classroom interaction system in upper elementary math and science classes for the past four years [2]. This paper reports on a study that investigated the ways in which a pen-based drawing tool could support 4<sup>th</sup> and 5<sup>th</sup> grade special needs students in learning multiplicative reasoning. The drawing tool is what we call a *stamp*, which enables students to draw an image, then duplicate the image to create a mathematical representation, e.g., four groups of six. We worked with a class of eight special needs students for 10 classroom sessions of between 45 minutes and an hour, using a structured sequence of multiplication and division problems. We identified several specific ways in which stamps helped the students gain more proficiency with multiplicative reasoning and based on pre- and post- assessments, saw evidence of their academic progress.

### **2. PROBLEM STATEMENT AND CONTEXT**

Asking elementary students to draw representations of mathematical situations is a common strategy for helping the students understand the meaning of basic operations. Teachers often suggest that students “draw a picture” to help them figure out what a problem is asking. Mathematics education researchers have studied children’s drawings in response to math problems and see such drawings as an important bridge between the specific and concrete objects in the problems—and the abstractions that underlie mathematical reasoning. Woleck [5], for example, notes that students often use drawings as mathematical manipulatives, similar to Unifix cubes or counters, creating a picture of a situation, then counting the objects in the picture. She notes that if students need to draw multiple copies of the same object, they often begin to simplify the drawing, leaving out unnecessary features that might have been important in an artistic context, but become cumbersome in a mathematical context. Woleck also regards students’ drawings as “springboards for the talking of mathematics,” a way for students to share their mathematical thinking and their further questions.

Fosnot and Dolk [1] examine the role drawing can play in students’ learning of multiplication and division. In multiplication, students need to be able to count groups, each of which contains multiple objects. This need requires students to make a major shift in perspective from looking at individual objects to focusing on the group of objects as an individual entity, a process called *unitizing*. Fosnot and Dolk describe drawings, either presented by teachers or created by students, that support this unitizing process by depicting objects grouped “naturally,” e.g., people sitting at tables of 6, games packed 10 to a box, or apples in boxes of 6. Similarly, Smith [4] describes how children drew multiple copies of an object (in this case, candy bars) to support their thinking about a division problem.

Drawing many identical images, however, can end up being a tedious distraction from the mathematical focus of a problem, and students often make unintentional errors by drawing one too many or one too few copies. A pen-based system has just the right affordances to simplify the creation of multiple copies of a drawn image. The INK-12 project has been investigating the use of a pen-based classroom interaction system in upper elementary math and science classes for the past four years [2]. As part of this project, we designed and implemented *stamps*, digital structures that could be used to create multiple, identical “stamped” images that we hypothesized would support students’ learning of multiplication and division concepts. This paper reports on the use of the INK-12 stamp technology in a special education class of 4<sup>th</sup> and 5<sup>th</sup> graders who were learning multiplicative reasoning.

The research reported in this paper is part of a broader research question about ways in which digital tools can enhance students’ ability to express their mathematical and scientific understandings through drawing. By providing students with the ability to easily make multiple copies of an image, our software, which we call Classroom Learning Partner (CLP) adds structure to the digital drawings students create, without taking away the creativity and ownership freehand drawing confers. Another advantage of this added structure is that it aids automatic interpretation of student drawings, as described in a companion paper Koile et al. [3].

### **3. METHOD EMPLOYED**

The classroom setup in which we investigated this tool is as follows: Students and teacher each had a tablet computer, and a tablet was connected to a projector. At the start of a lesson, students logged in and were automatically connected to a wireless peer-to-peer network. They loaded the lesson’s electronic notebook, which had been created using an authoring system we have implemented. Students worked through the lesson and wirelessly submitted their responses to the teacher’s machine. Students used the pen to write number sentences, draw on a stamp, and make copies of the drawn image using a “handle” on the stamp (see illustrations below). Details of the design and use of the stamp tool, as well as teachers’ viewing of student responses, are described elsewhere [3].

The results reported here are from 10 sessions of between 45 minutes and an hour in a class of eight 4<sup>th</sup> and 5<sup>th</sup> grade students with disabilities in a working-class suburb of Boston. Students were assigned to this class based on having IQs below 70 as determined by district and/or state tests. The class included two students who had limited knowledge of English and others who had difficulties with focus and attention. The teacher had been working with the students on their multiplication facts, but did not have the curriculum resources to work on building conceptual understanding of the operations. Accordingly, we designed a series of lessons that focused on the idea of multiplication as an operation on equal groups, using stamps as one way to visualize groups. In creating students’ notebooks, we had the option of either including pre-made stamps, as in Figures 1 and 2 below, or a blank stamp on which the student would draw, as in Figures 3 through 8. In general, we used more pre-made stamps early in the lesson sequence as a way to scaffold students’ thinking about groups, and let students draw their own stamps in later lessons.

Pre- and post-assessments with multiplication and division items were administered, both orally and in written form, to all the students in this class. Items included multiplication and division computation, solving story problems, and problems involving visualization, e.g., If there are 3 tennis balls in 1 can, how many are there in 4 cans?

#### 4. RESULTS AND EVALUATION

Comparison of pre- and post- test results, described in more detail at the end of this section, showed improvement in students' understanding of multiplication, which we considered significant considering both the limited amount of time we spent with the students and their relatively low math achievement levels. The bulk of this section illustrates the ways in which students used the two varieties of stamps—pre-made and hand-drawn—to solve multiplication and division problems. We describe the following ways in which the stamps contributed to students' ability to create representations, which in turn supported the development of their multiplicative reasoning.

1. Students with fine motor control issues were less frustrated because they only had to draw one image and could erase without leaving a mess.
2. Students perseverated less on details of the drawings, since they only had to draw once.
3. Stamps provided a pre-defined, limited space in which students were to draw.
4. Students could first create, then move stamped images, organizing their representations as they went along.
5. Pre-drawn stamps modeled unitizing.
6. Students created unitized drawings on stamps, modeled on pre-made examples.
7. Drawing in a stamp was a clear first step for problem-solving, especially useful for these students, who often struggled with how to begin working on a problem.

In the first lesson, students used a pre-made stamp, with the object in the story already drawn. The first stamps students encountered explicitly modeled unitizing by including the group of objects to be counted on the stamp, as in Figures 1 and 2 below. The group consists of 4 cookies; it is further made into a single “object” by being on a single plate. Most of the students were easily able to create a well-organized picture of 6 plates of cookies, then use it to count the total number of cookies. Mickey,<sup>1</sup> whose work is shown in Figure 1, counts by 2's to solve the problem. Other students, such as Eliza, used the stamps and repeated addition to solve the problem, as shown in Figure 2.

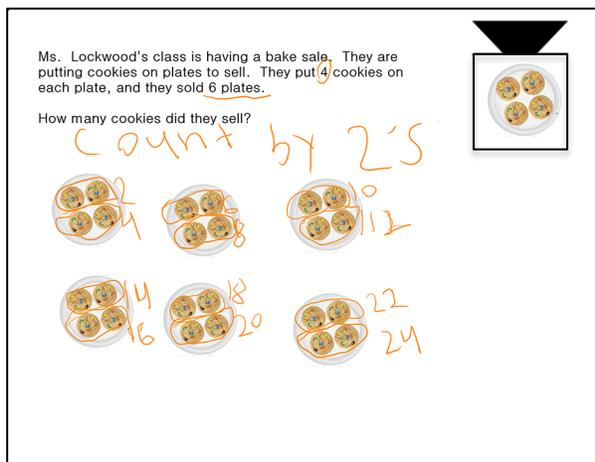


Figure 1. Mickey's work using stamps for 4 x 6

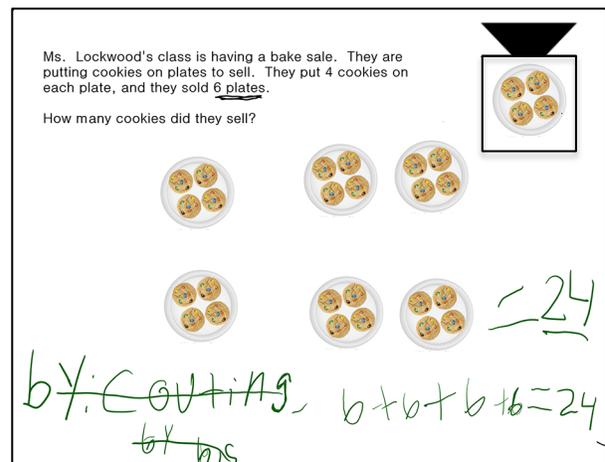


Figure 2. Eliza's work using stamps for 4 x 6

<sup>1</sup> All student names are pseudonyms.

After students had some experience with pre-made stamps, problems were introduced in which students drew their own image on the stamp. The students were highly engaged by being able to make their own representations. The teacher reported that in addition to engaging the students, the stamps helped students organize their representations and kept them from perseverating over details of the drawing. She also noted that, while these students often had trouble deciding what to do when presented with a word problem, drawing their own stamp provided a useful first step. Not only did it give them something specific to do, but the act of drawing the stamp required them to think about the quantities in the problem in a way that seemed to prepare them for the next step—that of creating multiple stamped images.

The students with a limited knowledge of English did particularly well when they were able to draw their own stamps. Hector created a stamp of a car with four wheels, made several copies of it and used repeated addition to solve a multiplication problem about wheels, as shown in Figure 3. When he had been asked a similar wheel problem orally on the pre-test, he had not been able to solve it. Hector also used a combination of a pre-drawn stamp (the person) and one he drew himself (the pencil), as shown in Figure 4, to solve a problem in which he had to group objects, rather than having them automatically “grouped” by virtue of being part of the same object, such as the car in Figure 3. Although he did not write the number sentence correctly (using the symbol for division instead of multiplication), he represented the problem correctly and knew what all of the parts meant.

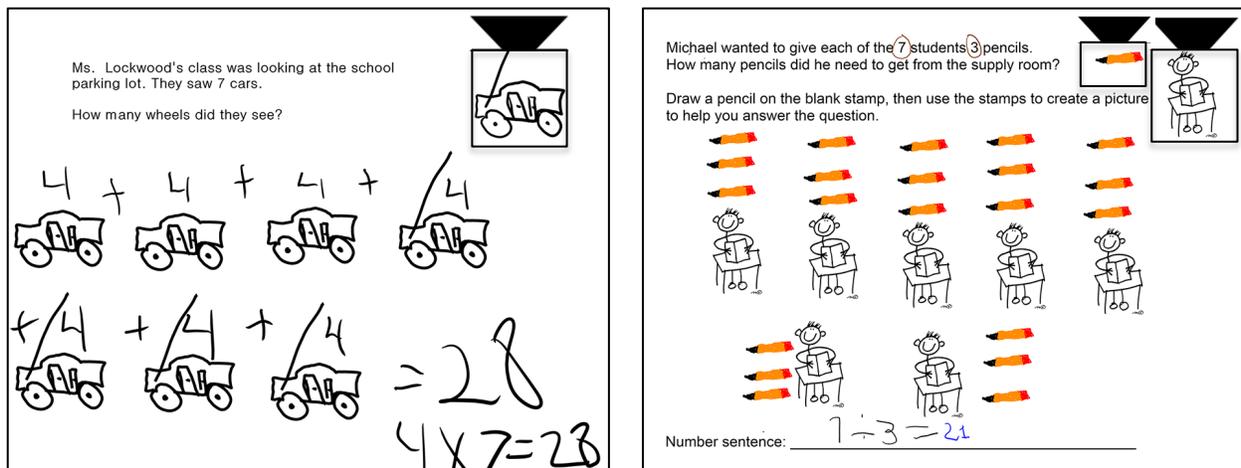


Figure 3. Hector’s work using a car stamp he drew Figure 4. Hector’s work using a pencil stamp he drew

Alfonso, another student with limited English proficiency, had more success drawing his own stamps, as opposed to using the pre-made stamps. He had some fine motor control difficulties with the pre-made stamps, which frustrated him, especially if he had to create a large number of stamped images. When he created his own stamp with 3 stickers on it for the problem, “If each student gets 3 stickers and there are 8 students, how many stickers are there in all?” he was able to represent and solve the problem correctly, as shown in Figure 5. Note that by creating a stamp with three stars on it, Alfonso seems to be showing some awareness of unitizing; he could have created a stamp with a single image on it (and some other students did). By creating a stamp with three stickers on it, he only had to create eight stamped images, rather than the 24 he would have had to create with a stamp of just one sticker. Alfonso also was able to use his drawings to correctly solve some division problems, as shown in Figure 6, even though when we started

working with the class, he did not understand what division was and was unable to answer any division questions on the pre-assessment.

There are 8 people in the class.  
Ms. Lockwood wants to give each person 3 star stickers.

How many stickers does she give out?

Use the blank stamp to create a picture to help you answer the question.



Star



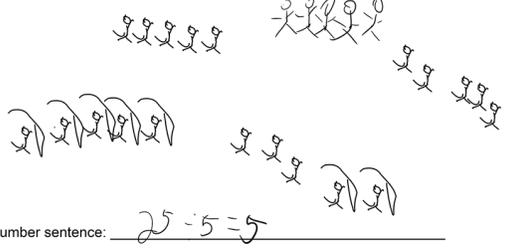
Ms. Lockwood gives out 24 stickers in all.

Figure 5. Alfonso's work with a drawn stamp

Nael wants to play basketball. There are 25 students.  
Each team has 5 students.

How many teams will there be?

Use the stamp to create a picture of the students. Draw a circle around each team of 5 students.

Number sentence: 25 ÷ 5 = 5

Figure 6. Alfonso's work on a division problem

For another student, Kordell, exposure to and practice with the stamps appear to have helped him organize his work. In the first example, Figure 7, he seems to have lost track of the number of cars whose wheels he is counting, and he arrives at the wrong answer. In the second example, Figure 8, which occurred after several days of using stamps, he was able to systematically organize the images for a more complex problem and arrive at the correct answer.

Ms. Lockwood's class was looking at the school parking lot. They saw 7 cars.

How many wheels did they see?

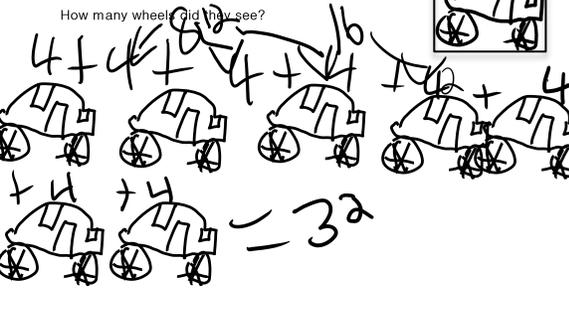
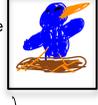



Figure 7. Kordell's incorrect work

Michael and Nael see 4 cats and 2 birds on the playground.

How many animal legs do they see in all?

Draw a cat stamp and a bird stamp. Then use the stamps to create a picture to help you answer the question.





They see 20 animal legs in all.

Figure 8. Kordell's correct work

The pre- and post-assessments suggest that the students from this special needs class made real progress in solving multiplication and division problems through their experience using CLP's stamps. On the post-assessment, they were more confident in their ability to solve problems. They all had a strategy for each problem, even if it was counting by 1's, and were more likely to explain how they got an answer than they had been on the pre-assessment. For example, on the post-assessment:

1. For  $20 \div 5$  one student said that she put cubes in her head, indicating that she put 20 in 5 groups of 4.
2. For  $4 \times 3$ , one student said, "I know  $3 \times 3$  is 9," and then he said, "10, 11, 12."

3. One student solved  $10 \div 2$  by remembering that  $5+5=10$  and reasoning that there were 2 groups of 5 in 10.
4. One student said “ $36 \div 6$  is 6 because  $6 \times 6 = 36$ .”

As there had been during the class sessions, there was considerable variability in students’ ability to connect representations with number sentences on the post-assessment. Although a few students were able to connect representations with number sentences on the post-assessment, others were not. For example, for  $7 \times 3$ , some students drew 7 things and 3 things, as opposed to 7 groups of 3, but had the correct number sentence. Despite this variability, and especially given the difficulties these students often have with mathematics, we believe the affordances of the stamps are worth further exploration with this population, as well as with non special-needs students.

## 5. FUTURE WORK

We are interested in supporting students’ growth in mathematical reasoning beyond the single-digit multiplication tables, so we are designing tools that enable students to visualize more complex multiplication problems. One such tool would let students draw and partition arrays. Array-based representations can be used to help students envision how to think about a problem such as  $24 \times 36$  as the sum of four simpler problems:  $20 \times 30$ ,  $20 \times 3$ ,  $30 \times 2$ , and  $4 \times 6$ . We will try out these new tools with non-special-needs students in the coming year. We also will continue working with special needs students on single-digit multiplication and division using stamps.

## 6. ACKNOWLEDGMENTS

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