**Using Machine Analysis to Make Elementary Students’ Mathematical Thinking Visible**

**Introduction**

Visual representations help students make sense of mathematical ideas. When students create their own representations, they demonstrate their thinking in a visible manner.

The INK-12 project has been studying elementary math students’ use of visual representations.

**Goals**

Understand student use of representations in multiplication and division.

Provide feedback to teachers about students’ thinking.

Machine analysis provides critical information not always visible in the final work.

**Machine Analysis Routines**

Based on a human coding scheme, analysis routines recognize characteristics of visual representations, use patterns, and implied mathematical thinking.

1. **Record Interaction History**
   - Record a sequence of low-level actions, e.g., adding ink strokes or objects.

2. **Create Semantic Events**
   - Identify object attributes, cluster and add semantics to ink strokes, identify abstract actions resembling human history codes.
   - Resulting semantic events describe process.

3. **Analyze Semantic Events**
   - Analyze semantic events to recognize salient use patterns and tag work with relevant analysis codes.

**Dataset**

Dataset is 8,470 pages of student multiplication and work from a 5-week trial with a class of 22 3rd graders; each page has a replayable history.

A subset, 264 pages, was used as a basis for the human coding scheme and machine analysis routines: 12 final assessment problems for each of 22 students.

**Assessment Notebook**

Representation use (on 264 pgs):
- Array: 69
- Bins: 14
- Number line: 133
- No representation: 78

Other analysis:
- Multiple representations: 21
- Answer before representation: 54
- Answer changed after representation: 17

**Robust machine analysis routines will enable analysis of the 8,470 pages of work in our data set, furthering our knowledge of how students’ mathematical thinking can be made visible.**