

# Using Machine Analysis to Make Elementary Students' Mathematical Thinking Visible

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## Introduction

Visual representations help students make sense of mathematical ideas. When students create their own representations, they demonstrate their thinking in a visible manner.

The INK-12 project has been studying elementary math students' use of visual representations.

Students use CLP, our tablet-based software, to create representations of their own choosing. They then wirelessly submit their work to the teacher, who can view the work in real time. Machine analysis can yield information about individual student representation use and learning during class or over time. It also can uncover trends in the class as a whole.

## Machine Analysis Routines

Based on a human coding scheme, analysis routines recognize characteristics of visual representations, use patterns, and implied mathematical thinking.



- Record a sequence of low-level actions, e.g., adding ink strokes or objects.
- Identify object attributes, cluster and add semantics to ink strokes, identify abstract actions resembling human history codes. Resulting semantic events describe process.
- Analyze semantic events to recognize salient use patterns and tag work with relevant analysis codes.

## Dataset

Dataset is 8,470 pages of student multiplication and work from a 5-week trial with a class of 22 3rd graders; each page has a replayable history.

A subset, 264 pages, was used as a basis for the human coding scheme and machine analysis routines: 12 final assessment problems for each of 22 students.

## Goals

Understand student use of representations in multiplication and division.  
 Provide feedback to teachers about students' thinking.

The student created an 8x8 array to represent the problem then skip counted by 8 to find the product (Image C). Analysis tags reveal that she recorded 8 and 16 (Image A), then used arithmetic for later numbers (Image B), suggesting that she may not know her 8 times table beyond  $8 \times 2 = 16$ .

Machine analysis provides critical information not always visible in the final work.

Analysis tags reveal that the student did not use the final number line (NL) (Image C) to find the answer: She wrote the answer after using a "dealing out" strategy with bins (Image A); created an array, suggesting that she was checking her work (Image B); erased everything, created the NL, then rewrote the answer.

This entire interaction history contains 341 low-level actions!

**Page 5**  
 Array use: 11 of 22 pgs  
 Array use strategies (on 11 pgs):  
 Skip counting: 7  
 Partial products: 2  
 Skip counting & partial products: 1  
 No array modifications: 1



**Extended Response** 3 points  
 Solve. Show how you find the answer.  
 12. A spider has 8 legs and a butterfly has 6 legs. How many legs do 4 spiders and 9 butterflies have in all?

**Page 13**  
 Representation use (on 22 pgs):  
 Array: 7  
 Number line: 7  
 Stamp: 3  
 No representation: 5

Robust machine analysis routines will enable analysis of the 8,470 pages of work in our data set, furthering our knowledge of how students' mathematical thinking can be made visible.

**Assessment Notebook**  
 Representation use (on 264 pgs):  
 Array: 69  
 Bins: 14  
 Number line: 133  
 No representation: 78  
 Other analysis:  
 Multiple representations: 21  
 Answer before representation: 54  
 Answer changed after representation: 17