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MACHINE INTERPRETATION OF STUDENTS' HAND-DRAWN MATHEMATICAL REPRESENTATIONS

Emerging Technology Research Strand

Kimberle Koile *MIT Center for Educational Computing Initiatives, kkoile@mit.edu*
Andee Rubin *TERC, andee_rubin@terc.edu*

1. ABSTRACT

The *INK-12: Interactive Ink Inscriptions in K-12* project is investigating the use of a pen-based wireless classroom interaction system in upper elementary math and science classrooms. This paper reports on progress made on the machine interpretation of students' drawings created using that system in learning multiplication and division. The problem addressed is that of finding the balance between freehand drawing and structured drawing, e.g., with pre-defined machine-readable icons. The innovation reported is what we call a *stamp*, which enables students to draw an image, then duplicate the image to create a mathematical representation, e.g., four groups of six. The stamp contains a hand-drawn image, but also creates a structured vocabulary that a machine can interpret. The resulting interpretation can be used to sort and group student work in order to help teachers in identifying students who need assistance and in choosing pedagogically interesting examples for class discussion.

2. PROBLEM STATEMENT AND CONTEXT

In the *INK-12: Interactive Ink Inscriptions in K-12* research project, we are continuing our investigation of the impact of a pen-based wireless classroom interaction system on teaching and learning in upper elementary mathematics and science [3]. *Pen-based* interaction enables students to create *inscriptions*, by which we mean handwritten marks, including text, drawings, and scribbles. Inscriptions have meaning and are especially important as visual representations in mathematics and science, where they play a key role in enabling students to articulate their understanding. In elementary math, these inscriptions are often in the form of drawings, which support student creativity and self-expression, engender ownership, and mediate between concrete and abstract [1, 5, 6]. (See [4] for further discussion.) *Wireless* communication supports easy and prompt sharing of representations, visual and otherwise, enabling teachers and students to explore alternate problem-solving strategies and ideas. Using our tablet-pc-based technology, which we call Classroom Learning Partner (CLP), students work in an electronic notebook, wirelessly submitting "pages" of their work to the teacher. The teacher can view student work in real time, identifying students who may need help and conducting class discussion based on example work that she can share publicly with the class. This sharing of student work and the resulting conversations about multiple representations and problem-solving strategies are critical to student learning. How, though, does a teacher choose pedagogically interesting examples for these conversations? And what happens when each student in a class of 25 decides to create several different representations, either to correct perceived mistakes or to suggest alternate strategies? The teacher gets a wealth of examples that could give her insight into her students' thinking, but as we have observed in classrooms, viewing and identifying pedagogically interesting work and gaining insight into what students know—or don't—can be challenging even with a small number of students. What if the student work, however, could be sorted into groups so that the teacher could easily see similarities and differences or responses that illustrate correct reasoning or common misconceptions? One of the goals of our research is to provide the

teacher with this kind of information by developing technology that can collect, but also interpret, group, and sort student work. Machine interpretation of handwritten text is possible, but interpretation of freehand drawings is extremely difficult. How could a machine, for example, "understand" that the student work shown in Figure 1 represents eight groups of three? One of the goals of our project is to investigate this question.

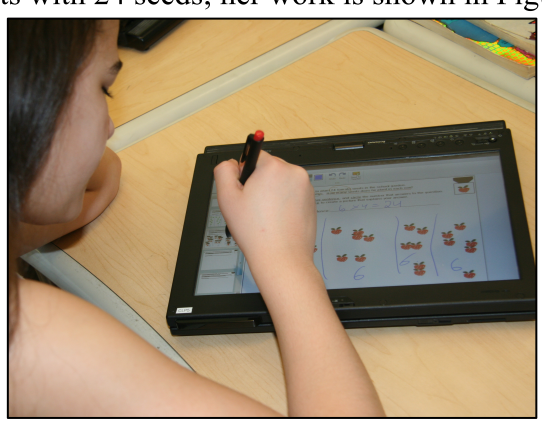
Figure 1. Student's multiplication problem

3. METHOD EMPLOYED

The key idea behind machine interpretation of a drawing is to add enough structure to enable the interpretation, but not so much that a student cannot still draw creatively. In upper elementary mathematics, there are many kinds of computational problems in which students may find it helpful to draw multiple copies of an image, such as in Figure 1. This observation led to the design and development of what we call a *stamp,* which enables students to draw then create identical images, much as a physical stamp does. The stamp creates a structured vocabulary, which a machine then can interpret.

Taking a design research approach [2], we prototyped new software features, trialed the features in classrooms, and used the results to inform redesign, then repeated the process. The classroom setup in the trials is as previously described [3]: Students and teacher each have a tablet computer, and a tablet is connected to a projector. At the start of a lesson, students log in and are automatically connected to a wireless peer-to-peer network. They load the lesson's electronic notebook, which the teacher has created using an authoring system we have implemented. Students work through the lesson—individually, in groups, or as a class—and wirelessly submit their responses, which appear on the teacher's machine and are stored on a local classroom server. The teacher views student work, circulating through the classroom identifying students who need help. She also conducts class discussion focused on student work examples that she has selected and sent wirelessly to the projector to be displayed anonymously.

The mathematics problems discussed in this paper focus on multiplication and division, a topic for which representations with multiple identical images, and hence stamps, are ideal. Shown in Figure 2a, for example, is a student using a stamp to "draw" a representation for a division problem that asks her to explain how many seeds would be in each of four rows if she starts with 24 seeds; her work is shown in Figure 2b.



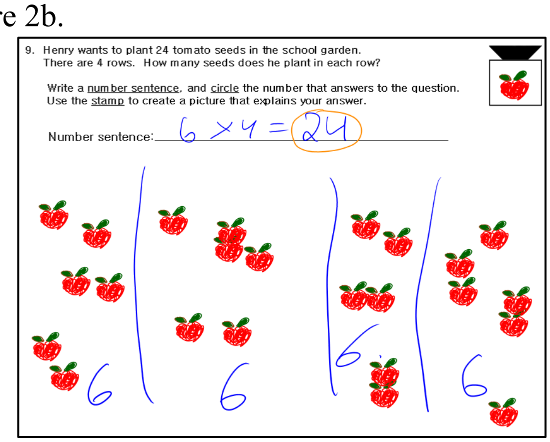


Figure 2. a. Student using stamps for division b. Problem shown at left: How many seeds per row?

Creating representations. To provide the stamp functionality, we devised a way for the pen to be used for three distinct functions: to draw an image, make copies of an image, and operate on the stamp object itself—move, delete, and resize. We separated the functionality in the following way: In the body of the stamp, the pen draws. The handle—a black trapezoidal region that resembles a physical stamp handle—is used for copying: Holding the pen down on the handle turns the region green to indicate that it has been "grabbed." Then dragging with the pen down causes a copy of the stamp to follow the path of the pen. Raising the pen in a final location places a copy of the stamp's image, much as using a physical stamp produces a copy of an image. Finally, operating on a stamp itself is achieved by means of hovering over a stamp's handle to pop up operators, as shown in Figure 3. On the copies of the stamp's image, students can draw with the pen or hover to pop up operators; they also can delete a copy by turning the pen over, using the button on the top of the pen as if using a pencil eraser.

Figure 3. Popped up icons for deleting, moving, resizing

Interpreting representations. In a mathematical representation containing multiple identical images, the identity and grouping of the images is critical to understanding the representation. We can see easily in Figure 2b, for example, that the images are organized into four groups of six and that each image represents one object, namely a seed. In order for a machine to produce this interpretation, it too needs to know how the images are grouped and what each image represents—both the syntax and the semantics of the images. Consider the examples in Figure 4. In the example on the left, a machine can identify that two different stamps were used and that the representation includes two copies of one and four of the other. It cannot tell, however, that one stamp represents two things (legs) and the other represents four. Similarly, in the example on the right, the machine can identify four copies of one stamp and three copies of a second one. In that example, we asked the students to write in a word for what each stamp represented. The student whose work is shown also wrote the number of things represented, which is the semantic information the machine needs in order to interpret the representation correctly as "four groups of four and three groups of eight," rather than simply "one group of four and one group of three." Note that supplying a word for what a stamp represents is not sufficient for enabling the machine to infer the number of things represented by a stamp. The problem on the right in Figure 4, for example, could be asking about a cat, rather than a cat's four legs. Rather than attempting to identify the stamp's drawing or using information supplied by a teacher, we will ask the student to supply information about what the stamp represents.

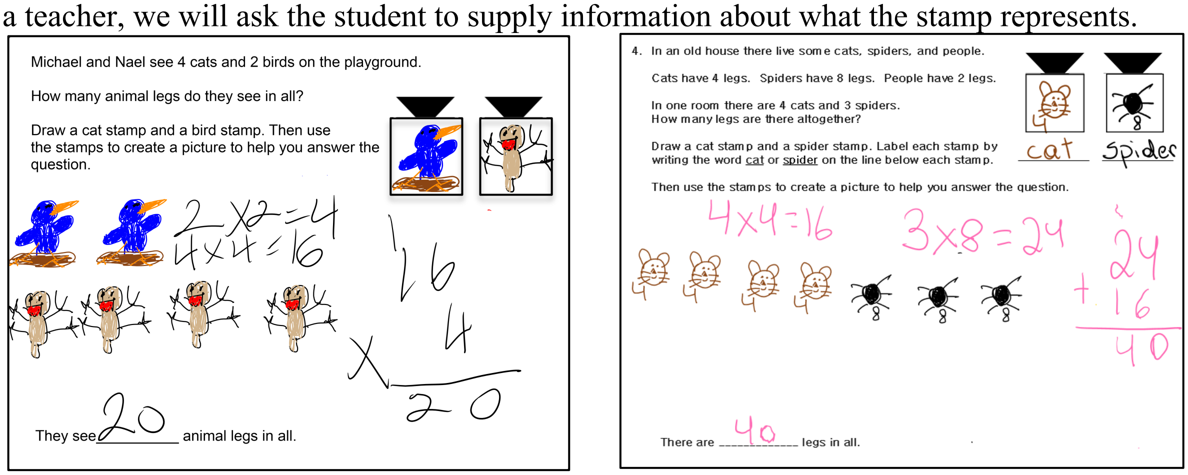


Figure 4. Student multiplication problem representations that use multiple stamps

Grouping in the Figure 4 examples above is achieved by having each stamp represent a group, e.g., a group of four legs. The examples in Figures 2 and 5 illustrate alternate ways in which students have created groups. In each example, a student has drawn an image on a blank stamp and created a representation using copies of that image. In Figures 5c and 5d the author supplied pre-made stamps—of a person and a box—to help students organize their thinking.

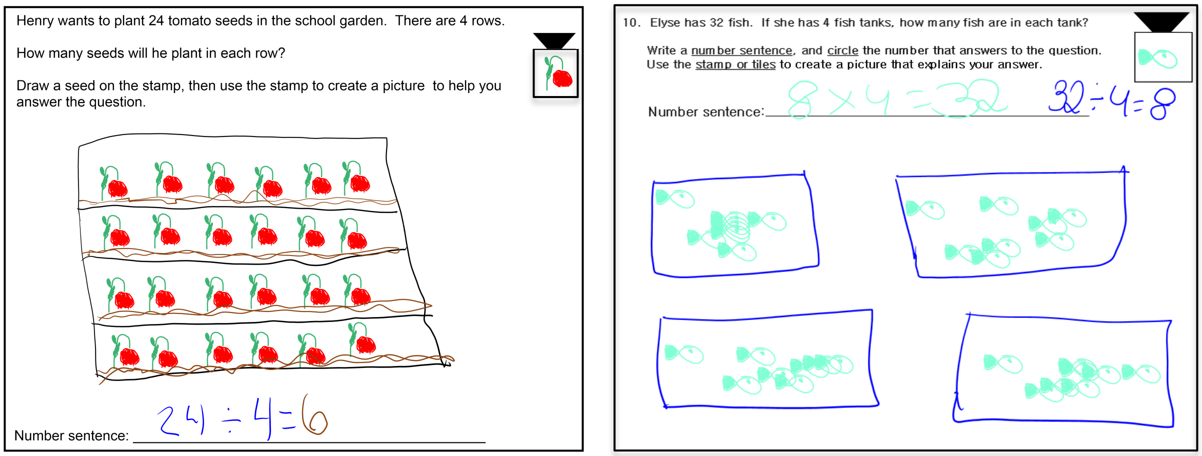
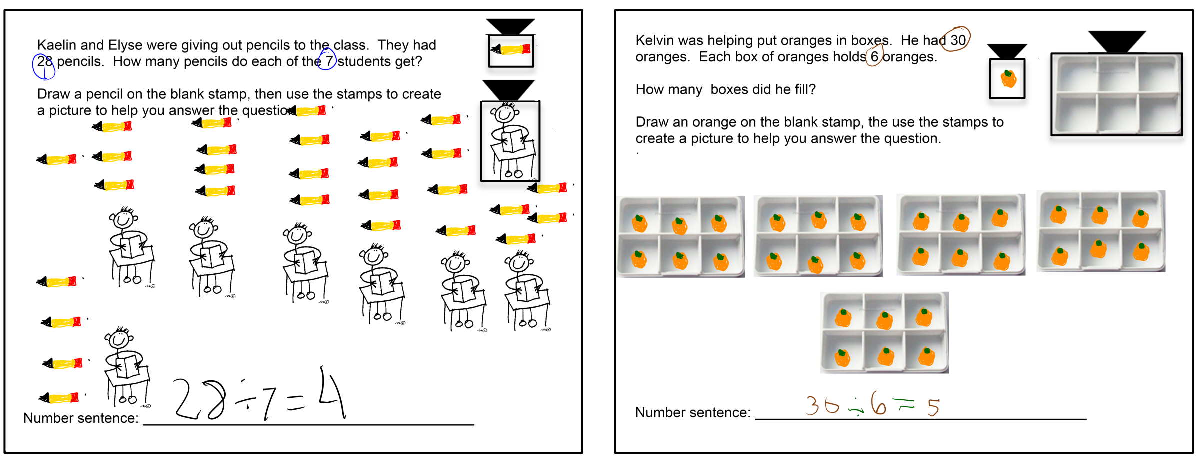


Figure 5. a. Grouping via contiguous ink *b.* Grouping via discrete ink shapes



c. Grouping via proximity *d.* Grouping via containers

Figures 2a, 5a, and 5b illustrate students using ink to group their stamped images via discrete lines, contiguous lines, and discrete boxes, respectively. In order for the machine to identify the groups, and return an interpretation such as "four groups of six," it must recognize the regions delineated by the ink and the stamped images within those regions. Figure 5b also illustrates a benefit of using stamped images: The overlapping ink strokes do not cause a problem for machine interpretation since the strokes are stored in separate stamped images. Figure 5c illustrates another method for grouping, namely grouping by image proximity. This example also illustrates a potential problem with this sort of grouping by machine: We can see that the student intended seven groups of four. Some of the stamped images, however, are equally close and could be considered part of more than one group, e.g., the four pencils on the right side might be considered part of a group of five that also included one pencil just to their left. Finally, the example in Figure 5d illustrates grouping with what we call "containers"—stamped images that serve to organize other stamped images. In this example, a student created a stamp for an orange, then used that stamp to fill in the boxes to represent five groups of six. As in previous examples, in order to produce this interpretation, the machine needs to be supplied information about what the stamp represents—a single object in these examples.

4. RESULTS AND EVALUATION

Creating representations

We have trialed the stamp functionality so far in two schools, with three teachers, 75 fourth and fifth grade students, over the course of 16 classroom sessions and 68 math problems that employed stamps. A typical classroom session lasted from 50 to 90 minutes, and we usually worked in the same classroom for three to five days at a time. We experimented with several different stamp designs, e.g., with different locations and shapes of stamp handles and means of creating stamped images. In one classroom session, we asked students to compare the use of the pen and the use of touch for creating copies of a stamp. Most students preferred the pen—they could more accurately position the stamped images by placing the pen on the handle and dragging the pen: The pen did not obscure the stamp as their hand did when using a finger to place an image. In addition, they liked not having to change the grip on the pen, as they did when using a pen for drawing and a finger for dragging the stamp.

Our current stamp design has been used to good effect. Teachers reported that stamps helped students stay engaged and focused on mathematical thinking. They liked that the stamps enabled students to draw, but without getting distracted or perseverating over details of their drawings. Students' multiplication work using stamps has shown clear organization and use of strategies, e.g., repeated addition. One special education teacher noted that the stamps appeared to be just the right amount of structural scaffolding for her students: It was clear how the students were to start (draw on the stamps) and what they needed to do next (create several copies). Indeed, for some of her students, extended practice with stamps appeared to facilitate significant growth in their multiplicative thinking. (See the companion paper [4] for details of a learning study conducted with this teacher and her students.)

Interpreting representations

Our interpretation routines are able to identify the location and number of particular stamps on a student work page. Given problems with multiple stamps, as in Figure 4, and information about what the stamps represent, e.g., four legs, the routines also can return an interpretation such as "four groups of four and three groups of eight." We are currently testing Microsoft's shape recognizer to see if it can be used to recognize ink lines and shapes that students use to demarcate groups. We are implementing and testing our container routines. Preliminary results indicate that if a stamp has been explicitly tagged as a container by the author, then identifying stamped images that overlay it, and that therefore can be considered part of a group, is fairly straightforward. If each group is understood to contain only one kind of image, then our routines have been able to deduce when a stamp plays the role of a container: If a single stamped image, for example, is overlaid with five images created from a different stamp, the machine can interpret the images as a group of five, without including the deduced container in the group. If, however, groups can contain images that are not all identical, containers must be tagged; the machine cannot deduce the correct grouping, but will return one group that may erroneously include what should be considered a container. We will continue to test and refine our grouping routines using work from classroom trials last spring, as well as new work from upcoming trials.

5. CURRENT AND FUTURE WORK

We have recently started working with a group of 12 students in an after-school program. We noted during our first trip there that several of the students seemed to have trouble hovering over the stamp handle in order to access the stamp operators. We plan to experiment with an alternate design that has a distinct region for accessing stamp operators, leaving the handle for only copying. We will experiment, for example, with students popping up stamp operators by tapping on a bar at either the top or bottom of the main body of the stamp. This design may prove easier for new users.

As mentioned earlier, we are currently evaluating our interpretation routines on representations created by fourth and fifth grade students using stamps for multiplication and division problems. In addition, we are working on an interface for students to use when entering information about stamp semantics, e.g., how many things a stamp represents and whether the stamp serves as a container. The work involves both technical considerations, e.g., whether to use handwriting recognition routines; and curriculum development, e.g., determining the best way to ask students what their stamps represent. Some students, for example, may not understand the distinction between an object and its parts, e.g., one bicycle vs two wheels; or may not be proficient with "part-whole" terminology.

We are working on clustering routines that will group student work based on interpretation of representations. We also are developing routines that will evaluate student work with respect to teacher-supplied representations. These clustering and evaluation routines will be integrated with the interface that supports viewing and sorting student work so that teachers, for example, can view all student work that exhibits a particular representational strategy.

6. ACKNOWLEDGMENTS

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