6.UAP Final Report: Identifying Bins-Dealing Strategies in Classroom Learning Partner

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I. Introduction

Rumor has it that Sir William Curtis, a British Member of the Parliament, once quipped that the three fundamental subjects of elementary education are the 3 R's: "reading, 'riting, 'rithmetic." Arithmetic in particular has seen its pedagogy change in the past few years thanks to the advent of tablet technology in classrooms. This trend is no surprise, as software that uses digital ink and wireless communication of said ink between students and their teacher can enhance K-12 education in ways inaccessible until recently. One particular tablet-based classroom interaction system called *Classroom Learning Partner (CLP)*, developed by Dr. Kimberle Koile and her research team at MIT, aims to facilitate STEM pedagogy by supporting the use of this digital functionality in creating, manipulating, and annotating visual representations, which are critical in STEM disciplines. In the NSF-funded project *INK-12: Teaching and Learning Using Interactive Ink Inscriptions in K-12* (ink-12.mit.edu), Dr. Koile and her team have focused on tapping the technological potential of tablets to help students in younger grades learn math, particularly multiplication and division [Koile & Rubin 2015, Koile & Rubin 2015a, Rubin et al. 2015]. CLP enables students to work in an "electronic notebook", with problems organized on "pages", and provides students with a combination of freehand drawing and digital tools to use in creating visual representations on the pages. In addition, CLP includes a machine analysis component whose goal is to "understand" students' visual representations in order to provide teachers with insights into student thinking [Koile & Rubin 2015b].

In this Undergraduate Advanced Project (UAP), we sought to improve the machine analysis capabilities of CLP. Specifically, we focused on machine analysis of students' visual representations created using a CLP tool called a *bin*, which is useful when teaching and learning

division. We will separate the discussion of our results into two sections: contributions related to new machine analysis software and evaluation of the new software.

II. Background

Out of all math topics, division poses a significant struggle for elementary school students, with long division being especially burdensome; as Koile and Rubin note, this struggle leads droves of students to the unfortunate conclusion that they are simply "bad at math" [Koile & Rubin 2015a]. Yet, understanding these topics is crucial, as understanding of division is an astonishingly powerful indicator for success in later math courses such as high school algebra [Siegler 2012]. To help teachers overcome this pedagogical challenge, CLP highlights important behavioral patterns in student work, such as evidence of misunderstanding or preference for certain problem-solving techniques; these patterns can then be used to structure lesson plans optimally. Prior to this UAP, CLP already had several tools for teaching division, such as arrays and number lines, and machine analysis tools to identify students' patterns of use with the tools. In a trial with third graders in a Boston-area classroom, CLP's bin tool showed significant potential for helping students understand division. The goal of this UAP project was to implement machine analysis tools for the bin tool.

CLP's bin tool can be described as follows. The student uses the interface to add or delete bins, which are objects with a closed boundary, by default, rectangles of fixed dimensions. Then, he or she "deals" marks into or remove marks from the bins. To "deal" means to add marks sequentially to bins until a desired number of marks has been added [e.g., Clements & Sarama 2004].

The CLP version of a mark is an object that students place on a page. To help students focus on math rather than bookkeeping tasks, a bin reporter at the top-right of the interface keeps track of how many marks have been dealt. The bin tool can be used to tackle division problems in a way that is beneficial to visual learners. Moreover, the exact strategy that the student employs in distributing the marks, which we call their *bins-dealing strategy*, is of particular interest because it can elucidate the student's thought process.

CLP's machine analysis routines are possible because CLP stores a *history* of a student's interaction with CLP's tools. The routines look for patterns in this history, identifying more abstract actions, which we call *steps*. Finally, the routines search the steps, looking for patterns that reveal something about a student's thinking, adding what we call analysis *tags*. The bins-dealing strategy is an example of such a tag and will be described below.

III. Software Contributions

To start, we implemented software routines that would search the history items in order to identify more abstract actions. In addition, we increased the flexibility of the routines so that they can recognize inkstrokes as marks. There are two advantages to enabling students to use ink instead of mark objects: erasing ink strokes is much faster than deleting mark page objects, and ink strokes are much smaller to store on a page than mark objects so loading and saving a page is much faster. Finally, we added routines that would search the steps for patterns revealing student problem-solving strategies, such as the bins-dealing strategy. The examples below describe the process in detail.

Shown in Figure 1 is an example of student work that illustrates the use of the bin tool for solving the division problem "64 ÷ 4." The student created four bins, representing the divisor,

then dealt out 64 marks, representing the dividend, with 16 marks per bin, representing the quotient. Also shown in Figure 1, on the right under the HISTORY tab, is a listing of the history of interactions for this example.

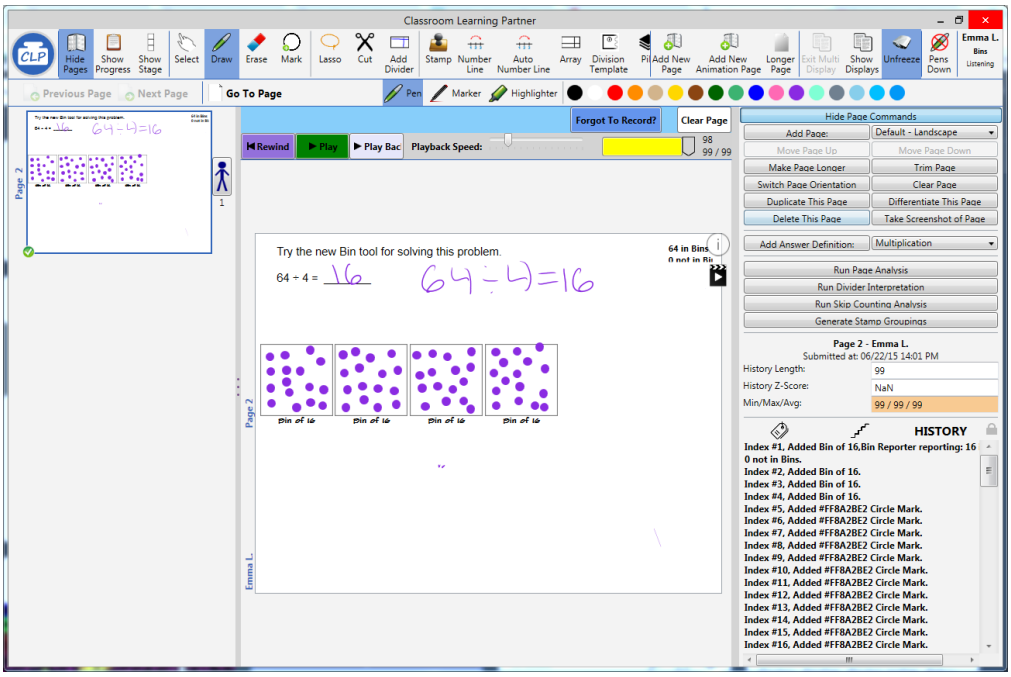

Figure 1: HISTORY tab for a bins representation of 64 ÷ 4

The STEPS tab in Figure 2 shows that the student dealt a single mark at a time into the bins following a bin 1, bin 2, bin 3, bin 4 pattern, indicating a deal-by one pattern. Each mark in this tab is accompanied by its color, shape, as well as the number of the bin it is dealt into (bins are numbered chronologically). This strategy, which is visible in the TAGS tab of Figure 3, can be more concisely summarized as “BINS deal [4 DB 1 D: 16],” which translates to creating 4 bins, dealing by 1, with a discrete mark, until a total of 16 marks are in each bin.

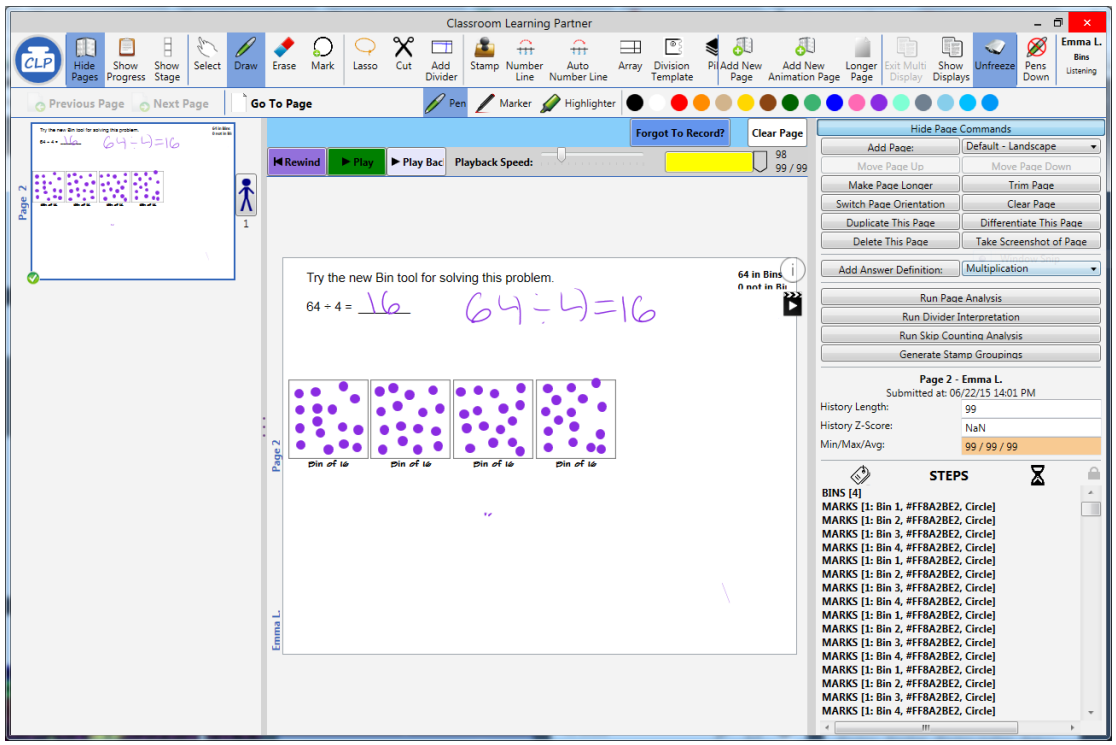

Figure 2: STEPS tab for a bins representation of 64 ÷ 4

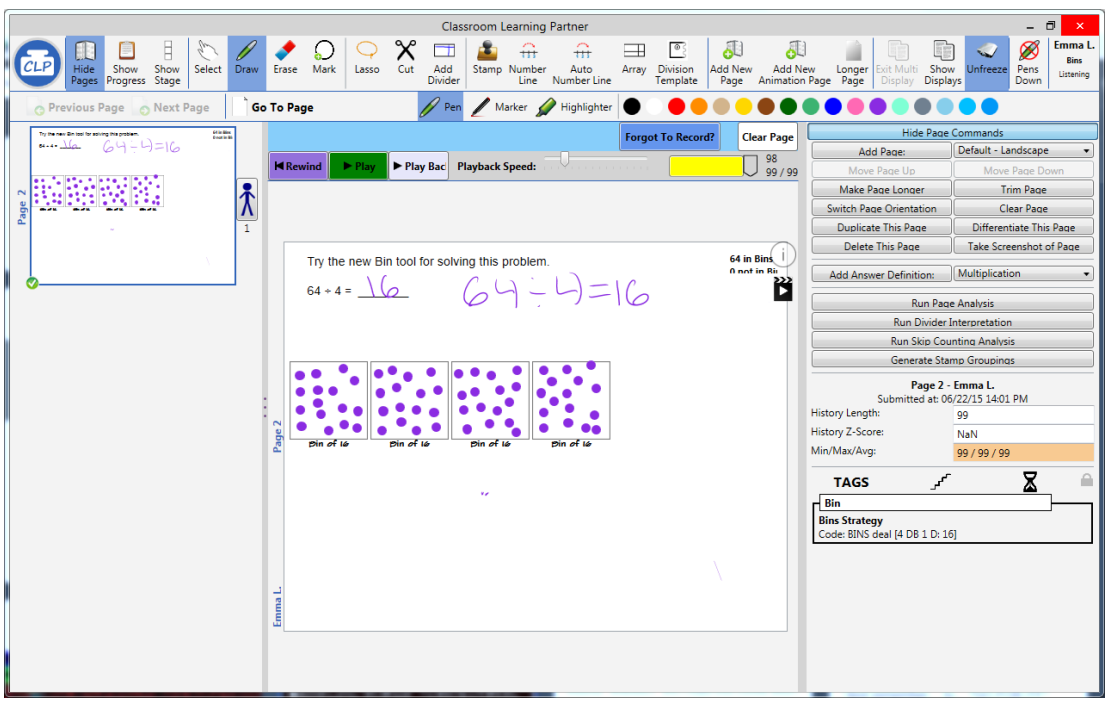

Figure 3: TAGS tab for a bins representation of 64 ÷ 4

To increase the flexibility of marks, we modified CLP's machine analysis to treat inkstrokes as marks if they are inside a bin. In the self-authored example shown in Figure 4, three marks of pre-defined shape are added to each bin, then three inkstrokes are added to each bin. Furthermore, in between some of the inkstrokes, arbitrary marks and inkstrokes are added outside the bins, in order to show that the software can ignore these irrelevant actions. The machine is able to recognize the strategy correctly by grouping the preset marks and the inkstroke marks together, while parsing out any objects dealt outside of bins. The strategy is correctly identified as dealing by three into three bins for a total of six objects per bin, which is abbreviated as "BINS deal [3 DB 3 D: 6]," which is shown in the TAGS tab in Figure 4. Figure 5 shows the corresponding steps; as one may guess, OUT denotes that a mark is made outside of bins and is therefore ignored during machine analysis. Such an occurrence is commonplace because students tend to make stray marks and leave them un-erased if the marks are not inside a bin.

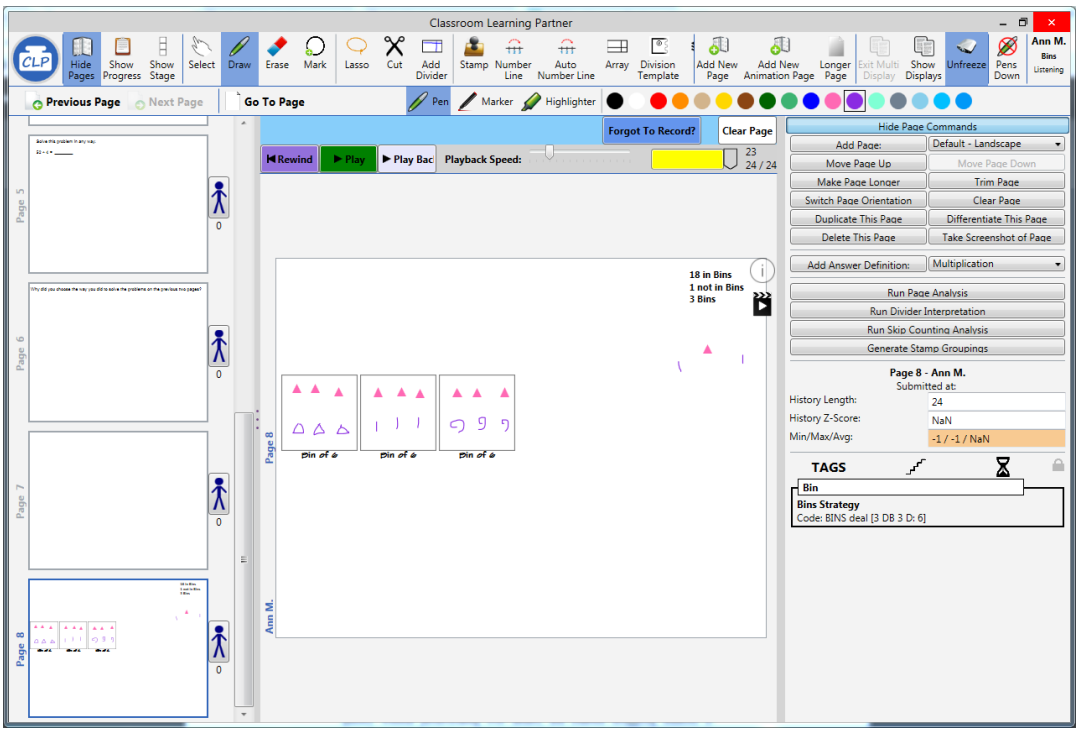

Figure 4: Deal-by 3 strategy shown in TAGS tab

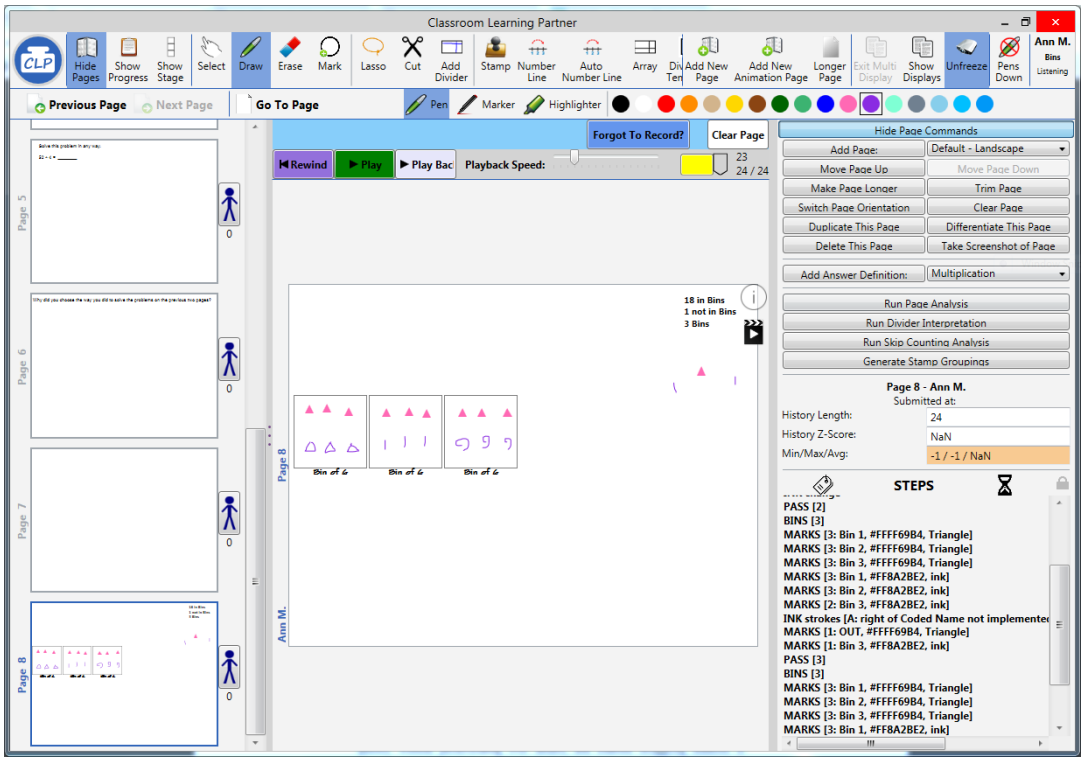

Figure 5: Deal-by 3 strategy STEPS tab

IV. Evaluation

After we had finished implementing the analysis routines for CLP's bin tool, we went through a sample of student work and evaluated how the machine analysis performed in comparison to human analysis. At this point, we reasoned that it would be useful to categorize student works based on *regularity*, which is defined as how closely a student follows a single "deal-by x" strategy, i.e., deals out consistently the same number of marks in each bin. We labeled each page of student work as regular, mostly regular, or irregular, depending on how closely the students followed their strategy. The first example of the "16 ÷ 4" problem, shown in Figures 1 through 3, is an example of regular work, because the same number of marks, namely one, is always dealt into bins in the order bin 1, bin 2, bin 3, bin 4. If a student's strategy on a given page closely followed a single strategy, but deviated from it at least once, we labeled it as mostly regular. If it deviated from it significantly, we labeled it as irregular. Naturally, this categorization can only have limited consistency because of human subjectivity, but it nevertheless sheds light upon important trends (see Future Work section).

Figure 6 shows an example of a page of student work that we labeled as mostly regular. This student computes "52 ÷ 4" by following a deal-by one strategy almost perfectly, but in the middle, he deal two marks into bin 1 instead of one. Later, he skips bin 1 to compensate for over-counting. Although the machine currently interprets this strategy as "BINS deal [4 DB 1 D: 13]," as shown in Figure 7, we seek something more nuanced to capture the correction that the student made.

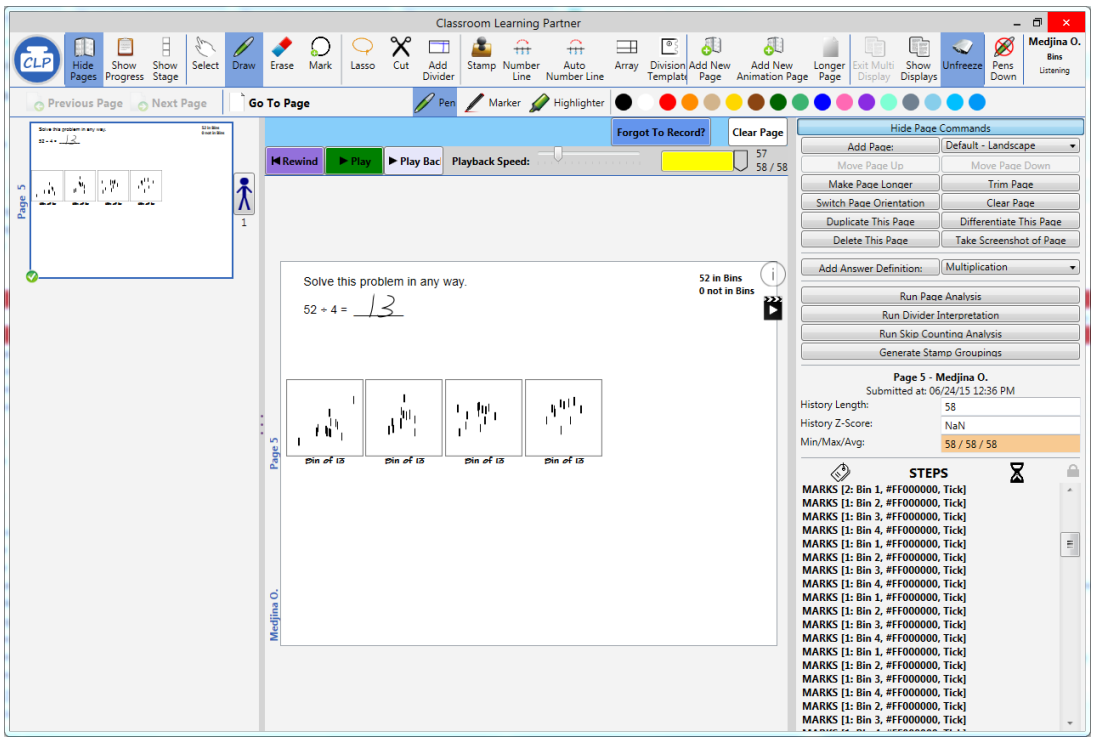

Figure 6: Slight deviation from deal-by 1 strategy, shown in STEPS tab

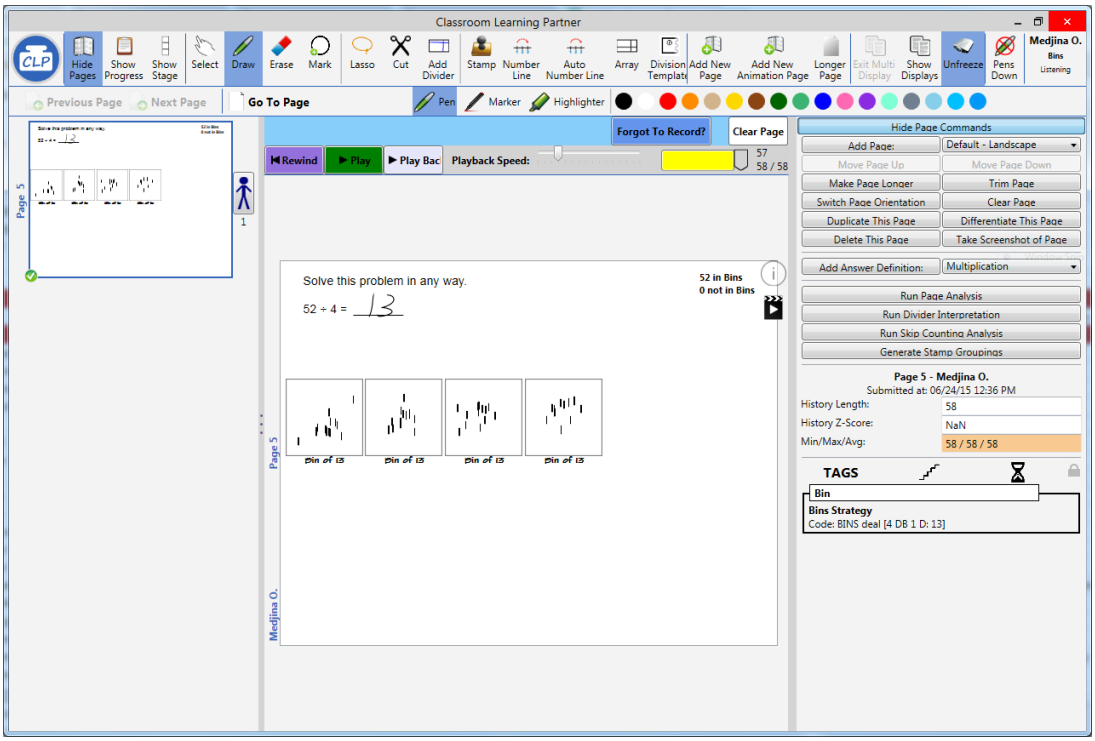

Figure 7: Deviation not captured, shown in TAGS tab

Irregular examples of student work are perhaps the most fascinating, since they exhibit the most variation. In the “78 ÷ 13”example shown in Figures 8 and 9, the student began with a deal-by one strategy, but switched to a deal-by four strategy for a single round of dealing in the middle. This change might indicate that the student had an intuition for how large the answer should be and provides insight into the student’s thinking. The machine currently recognizes this as “BINS deal [13 DB 1 D: 6]” because it follows the strategy first used. (See Figure 10.)

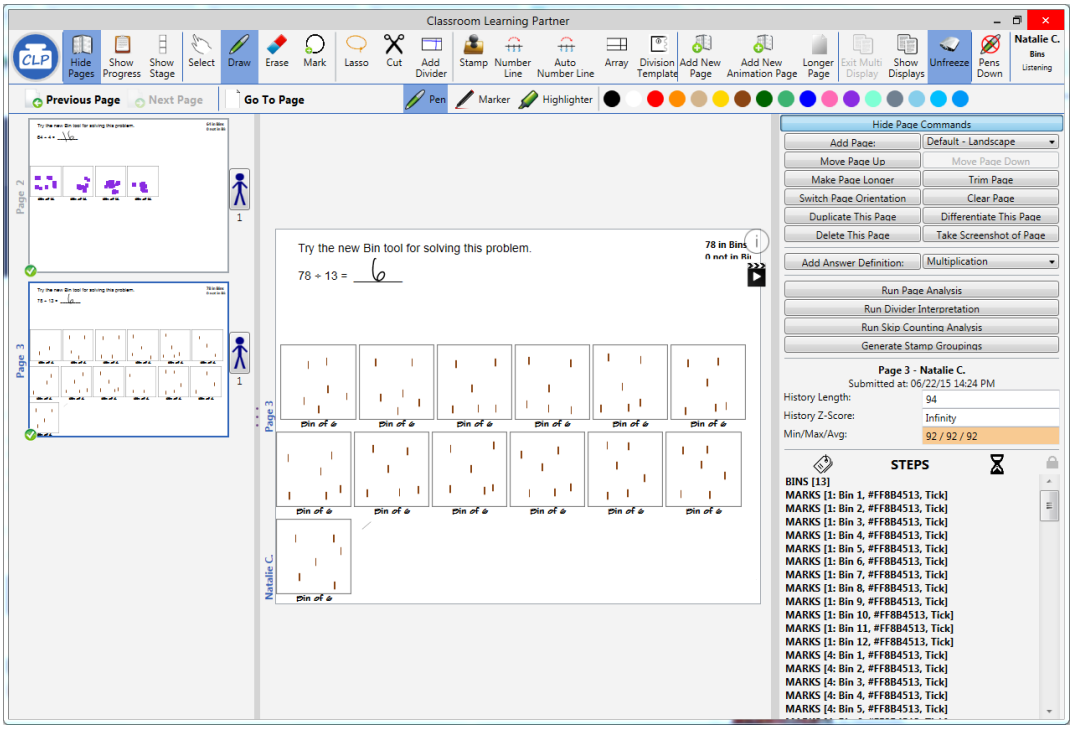

Figure 8: Change of strategy shown in STEPS tab

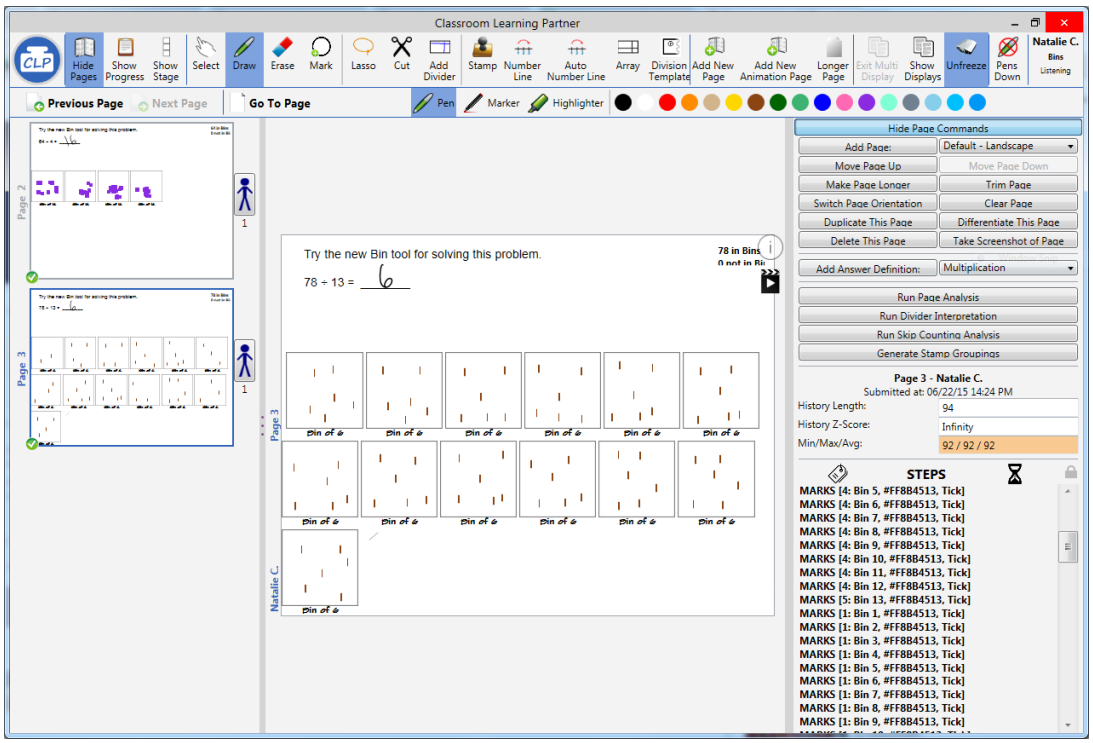

Figure 9: Change of strategy shown in STEPS tab (continued)

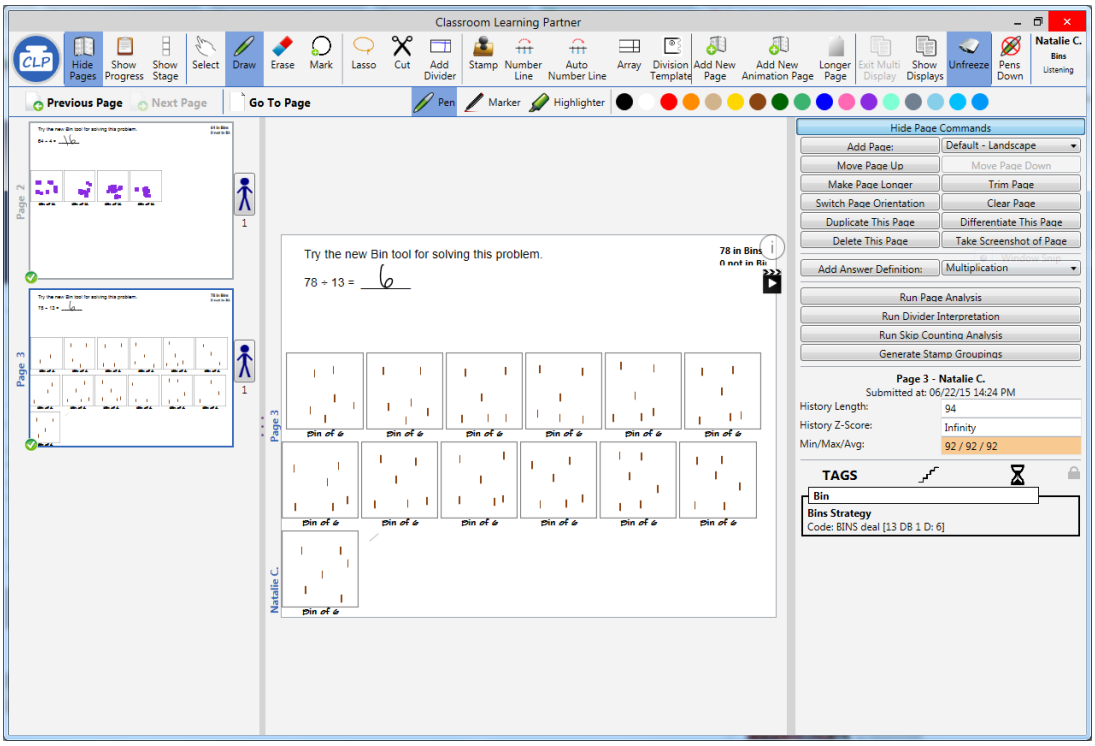

Figure 10: Change of strategy not captured, shown in TAGS tab

There are more unusual examples. Students may not distribute marks evenly, often due to counting errors. In the work shown in Figure 11, the student attempted the “78 ÷ 13” problem by dealing six into all but two bins: into those two bins, the student dealt seven and five respectively. Their strategy was summarized by the algorithm, as shown in Figure 11, as “[13 DB 1 D: 6, 6, 7, 6, 6, 6, 6, 6, 6, 6, 6, 6, 5]”. The analysis of the final configuration is correct, but since the bins contents are listed from left to right for each row of bins, the analysis does not reveal that the student was marking the bins right to left in the first row, then left to right in the second row, so that the seven and five marks occurred sequentially. Furthermore, the student left a mark outside to indicate a remainder of one in his answer, which although incorrect, would be interesting to capture. (Recall that, at present, all outside marks are considered stray and ignored during strategy interpretation.)

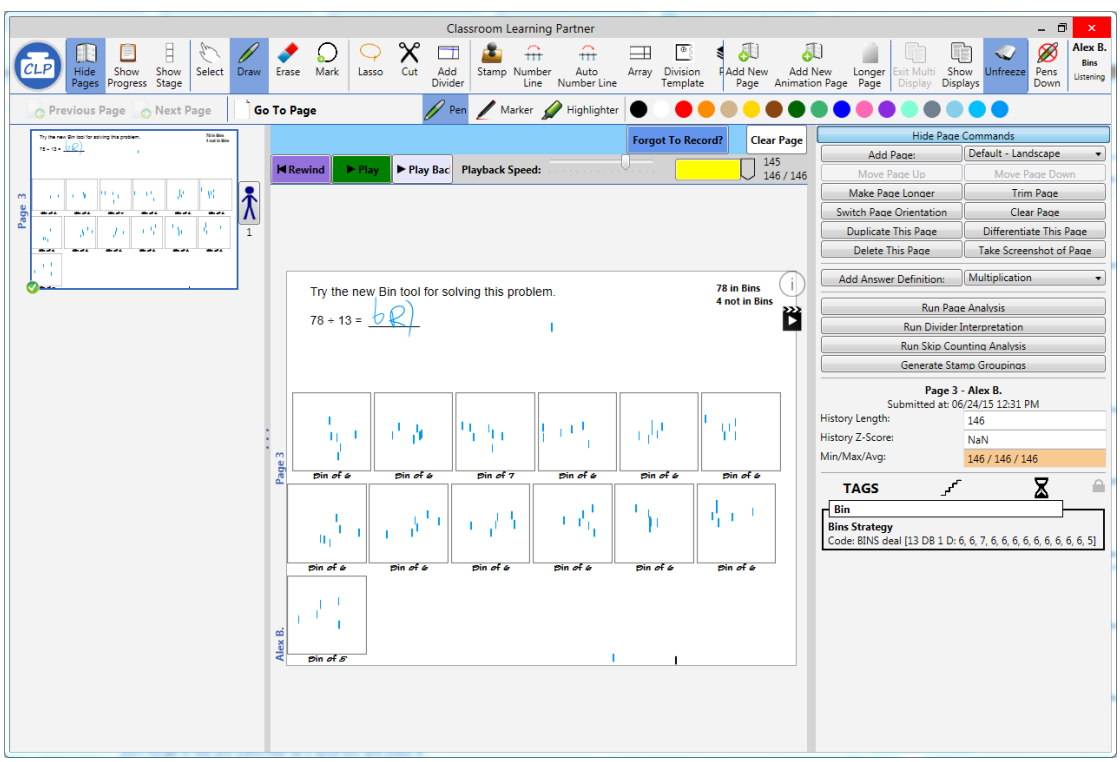

Figure 11: Uneven distribution across bins in TAGS tab

In total, we sampled 54 pages of student work. Below is the regularity distribution, according to human judgment, that we found:

Regularity	Regular	Mostly Regular	Irregular
Number of pages	15	22	17
Proportion of pages	28%	41%	31%

The machine analysis algorithm used to populate the BINS tag performed well on pages of student work with perfect or nearly perfect regularity: It was able to correctly reproduce the analysis for the 28% of pages denoted as regular, and reproduce the strategy in the analysis for over half of the 41% of pages denoted as mostly regular and reproduce the strategy for half of the 41% of pages denoted as mostly regular. We expect the latter statistic to increase as we continue testing and refining CLP's algorithm.

V. Future Work

As evidenced by our analysis of student work, the primary area of improvement for CLP's bin analysis is increasing the space of possible bins-dealing strategies to allow for nuance. Currently, machine analysis only shows a high accuracy on regular student work. Thus, consistent and concise notation needs to be defined and implemented for strategies that are mostly regular, which comprise the plurality of the work samples, or irregular. For example, "BINS deal [4 DB 3 D: 6; DB 1 D: 8]" might denote that a student dealt by three until each of four bins contained six marks; subsequently, the student dealt by one until each bin contained eight marks. However, this hybrid strategy is clearly contrived for convenience, and cleverer notation will be needed for highly irregular strategies.

Additionally, allowing for bins to be hand-drawn in ink rather than solely created by the push of a button may prove useful. This capability would give students an alternate way to create bins, though implementation may prove tricky: If, for instance, the student draws bins *and* marks as ink circles, then it will be necessary to distinguish the bins from the marks within. Furthermore, bins drawn in this fashion need to be distinguished from arithmetic writing, such as large zeros. Successful analysis of hand-drawn bins will depend on the accuracy of shape recognition algorithms to identify the bins. The advantage of the bins tool is that shape recognition is not necessary. In addition, the bins tool can help students keep track of the number of marks they have made—losing count of marks is a common problem that we witnessed in classrooms before we implemented the bins tool. Testing of the trade-offs between CLP's bins tool and hand-drawn bins will prove useful.

VI. Conclusion

Ultimately, division is difficult for teachers to teach, just as it is difficult for students to learn. As a consequence, software such as CLP has the potential to drastically improve math education by revealing teaching insights that only machines may be able to detect. It is our hope that our code development and our analysis in this UAP will one day help young students realize that they are not "bad at math" after all.

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